MATH GRADE 7 UNIT 3

CONSTRUCTIONS AND ANGLES

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LESSON 1: PAPER FOLDING

- Write your wonderings about constructions and angles.
- Write a goal stating what you plan to accomplish in this unit.
- Based on your previous work, write three things you will do differently during this unit to increase your success.
1. Which angles are adjacent? (There may be more than one correct answer. Check all that apply.)

A Angles 1 and 2
B Angles 2 and 4
C Angles 1 and 4
D Angles 3 and 4

2. Two angles are adjacent, and each has a side that lies on the same line. If one angle measures 35º, what is the measure of the other angle?

A 35º
B 55º
C 145º
D Cannot be determined

3. Which angles are complementary?

A Angles 1 and 2
B Angles 2 and 3
C Angles 3 and 4
D Angles 1 and 4
LESSON 2: ANGLES

4. Two lines intersect and one of the four angles formed measures 45°. What are the measures of the other three angles?

5. Explore the sketch shown. What is true about angle 1 and angle 2? Click and drag the blue dot to move the right angle.

Challenge Problem

6. Two lines intersect and one angle is four times as large as another angle. What are the measures of the four angles?
1. In a parallelogram, the diagonals are congruent but not perpendicular to each other. What is the most specific name for this figure?
   A Quadrilateral
   B Parallelogram
   C Rectangle
   D Square

2. In a parallelogram, one angle measures 60º. What are the measures of the other three angles?

3. In a parallelogram, one angle measures 90º. What are the measures of the other three angles?

4. What properties are shared between a parallelogram and a rectangle? What properties could be different between a parallelogram and a rectangle?

5. Two angles with the same measure and opening in the same direction are drawn using the same line as one side of each angle. What is true about the rays of each angle?
6. This parallelogram has angle measures of 120° and 60°.

At any of the four vertices of this parallelogram, there are three more angles outside of the figure. Which value could be the sum of these three angle measures?

A 90°
B 120°
C 270°
D 300°
LESSON 4: PROPERTIES OF QUADRILATERALS

1. A quadrilateral is found to have congruent and perpendicular diagonals. What is the most specific name for this figure?
   - A Square
   - B Rectangle
   - C Rhombus
   - D Parallelogram

2. A quadrilateral has two right angles and a 40° angle. What is the measure of the fourth angle?
   - A 40°
   - B 60°
   - C 120°
   - D 140°

3. A quadrilateral has the following angle measures, 45°, 60°, and 110°. What must be the remaining angle measure?
   - A 35°
   - B 45°
   - C 110°
   - D 145°

4. The diagonals of a quadrilateral are 3 in. and 5 in. Each diagonal divides the other diagonal into two congruent segments. What could this figure be?

5. In a quadrilateral, all four angles are congruent. If the diagonals are perpendicular to each other, what is the figure?

Challenge Problem:

6. Create a quadrilateral that has diagonals that are perpendicular to each other. What specific type of figure must this be?
1. The angle measures of two angles in a triangle are 50° and 100°. What is the measure of the third angle?
   - A 30°  
   - B 50°  
   - C 100°  
   - D 210°

2. If all three angle measures are given, under what conditions could a triangle be constructed? How many triangles could be made?

3. If two sides of a triangle are given, how many triangles can be formed? If the angle between the sides is also given, how many triangles can be formed?

4. What are the measures of angle 1 and angle 2?

   ![Diagram of a triangle with angles 48°, 117°, and 117°]

   - Angle 1: 48°  
   - Angle 2: 117°

Challenge Problem

5. \( \triangle DEF \) is an equilateral triangle, and \( \triangle EFG \) is an isosceles triangle. What are the three angle measures of \( \triangle DEG \) and what type of triangle is it?

   ![Diagram of a triangle with unknown angles]
EXERCISES

1. The angle measures of two angles in a triangle are both 45º. What type of triangle is it? (There may be more than one correct answer. Check all that apply.)
   A  Acute triangle
   B  Right triangle
   C  Scalene triangle
   D  Isosceles triangle

2. You are told a triangle has only two congruent angles. Which statement must be true about its sides?
   A  All three sides must be the same length.
   B  All three sides must be different lengths.
   C  Two of the sides must be the same length.
   D  Two of the sides must be shorter than the third side.

3. You are told a triangle has three congruent sides. Which statement must be true about its angles?
   A  All three angles are different measures.
   B  All three angles are the same measure.
   C  Two angles are the same measure, and the third angle is larger.
   D  Two angles are the same measure, and the third angle is smaller.

4. Which statement must be true of three side lengths in order for them to make a triangle?
   A  All three side lengths must be different lengths.
   B  All three side lengths must add up to 180.
   C  The sum of the lengths of the two shorter sides must be less than the length of the third side.
   D  The sum of the lengths of the two shorter sides must be greater than the length of the third side.
5. You know two of the side lengths of a triangle are 13 cm and 15 cm. Which of the following side lengths could be used for the third side? (note, more than one correct answer, select all that apply)

A 5 cm  B 13 cm  C 27 cm  D 30 cm  E 38 cm

Challenge Problem:

6. You are given three sticks, measuring 5 cm, 7 cm, and 15 cm. Can you arrange the sticks on the ground to form a triangle? Explain why or why not. Draw a sketch to support your explanation.
1. What is the sum of the angle measures of a heptagon (7 sides)?
   A 900°  B 1,260°  C 1,620°  D 1,800°

2. A regular polygon has angle measures of 135°. What type of polygon is it?
   A Pentagon  B Hexagon  C Octagon  D Decagon

3. A pentagon has three right angles. What could the other two angles measure?
   A 90° each  B 130° and 140°  C 20° and 150°  D There is not enough information.

4. If a parallelogram is divided by its two diagonals, is the angle sum of the parallelogram 720°? Explain your reasoning.

5. A classmate measures all of the angles in a polygon and gets a total of 2,400°. However, the angle sum is not correct. Show why the sum is not correct and what the correct angle sum might be.
Challenge Problem

6. For each regular polygon shown, the diagonals meet in the center and divide each other into two equal segments.
   a. What is the measure of each angle at the center of the polygon?
   b. As the number of sides increases, what is that angle measure getting closer to?
EXERCISES

1. The diagonals of a quadrilateral are perpendicular to each other and divide each other into equal segments. What is the most specific name for the figure?
   A Parallelogram
   B Rectangle
   C Trapezoid
   D Rhombus

2. The known angle measures in a pentagon are 95°, 105°, 90°, and 120°. What is the measure of the last angle?
   A 90°  B 110°  C 120°  D 130°

3. Solve for \(x\).

4. Solve for \(x\).
5.  a. In the regular octagon shown, find the measure of angle 1.
b. What figure is inside the octagon? How do you know?

Challenge Problem

6. $\triangle ABC$ and $\triangle BCD$ are isosceles triangles. Angle $CAB$ measures $75^\circ$ and angle $ABD$ measures $45^\circ$. What are the measures of angle 1, angle 2, angle 3, and angle 4?
LESSON 9: PUTTING IT TOGETHER

• Read through your work on the Self Check task and think about your other work in this lesson.

• Write what you have learned.

• What would you do differently if you were starting the Self Check task now?

• Record your ideas. Keep track of any strategies you have learned.

• Complete any exercises that you have not finished from this unit.
MATH GRADE 7 UNIT 3

CONSTRUCTIONS AND ANGLES

ANSWERS FOR EXERCISES
1. A Angles 1 and 2  
   D Angles 3 and 4  
   Both pairs of angles share a common side, and they all share the same vertex.

2. C 145º  
   Since the angles are adjacent and have sides that lie on the same line, they must be supplementary angles.  
   35º + 145º = 180º

3. D Angles 1 and 4  
   The measures of complementary angles add up to 90º.  
   65º + 25º = 90º

4. 45º, 135º, 135º  
   The angle opposite the 45º angle must measure 45º as well because it is a vertical angle. (Vertical angles are congruent.) The angle adjacent to the 45º angle and the 45º angle are supplementary angles because each has a side that lies on the same line. The angle adjacent to the 45º angle must measure 135º.  
   45º + 135º = 180º

Notice that the measures of all four angles add up to 360º  
(45º + 135º + 45º + 135º = 360º).

5. Possible answer: Since the three angles form a straight angle, the sum of the angle measures must be 180º. The measures of angle 1 and angle 2 must add up to 90º because the other angle is a right angle (90º), and 90º + 90º = 180. So, angle 1 and angle 2 are complementary angles.
Challenge Problem

6. The measures of the four angles are 36°, 144°, 36°, and 144°.

The two angles that are described must be supplementary. (If they were vertical, they would be congruent.) If $x$ is the first angle, the second angle is $4x$:

\[ x + 4x = 180° \]
\[ 5x = 180° \]
\[ x = 36° \]

Second angle:

\[ 4x = 4(36°) = 144° \]

So, one angle measures 36°, and the second angle measures 144°. The other two angles have the same measures because they are vertical to the two angles.
1. **C**  Rectangle

   If the diagonals in a parallelogram are congruent, the figure must be a rectangle. A square is a rectangle, but it is also a rhombus and its diagonals would be perpendicular.

2.  The other three angles measure 60°, 120°, and 120°.

   Since the figure is a parallelogram, opposite angles are congruent and adjacent angles are supplementary. There will be a 60° angle opposite the known angle, and 120° angles next to the known angle (60° + 120° = 180°).

3.  The other three angles must all measure 90°. Since the figure is a parallelogram, opposite angles are congruent and adjacent angles are supplementary. There must be a 90° angle opposite the known angle and 90° angles next to the known angle, because they must add up to 180°.

4.  Parallelograms and rectangles both must have four sides, with two pairs of opposite sides that are congruent. The two pairs of opposite sides are also parallel. The four interior angle measures add up to 360°.

   Parallelograms differ from rectangles in that they have two pairs of congruent angles, while a rectangle has four 90° angles.

5.  The rays will be parallel. Each ray is moving in the same direction with respect to the line, so they will never intersect.

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**Challenge**

6.  **D**  300°

   The measure of the angle in the parallelogram and the measures of the three angles outside the figure add up to 360°.

   For the 60° angle, the other three angle measures can be found by subtracting its measure from 360°: 360° – 60° = 300°.

   For the 120° angle, the other three angle measures can be found by subtracting its measure from 360°: 360° – 120° = 240°. This is not an answer choice.
LESSON 4: PROPERTIES OF QUADRILATERALS

1. **A Square**
   A square has diagonals that are congruent and perpendicular.

2. **D 140°**
   The sum of the angle measures in a quadrilateral is 360°.
   \[
   90° + 90° + 40° = 220° \\
   360° – 220° = 140°
   \]

3. **D 145°. All four angles of a quadrilateral must add up to 360°.**
LESSON 4: PROPERTIES OF QUADRILATERALS

4. The figure would be a parallelogram because the diagonals bisect each other. More specifically, it could also be a rhombus if the diagonals happened to be perpendicular. The figure cannot be a rectangle because the diagonals are not congruent.

5. Since all four angles are congruent, they must each measure 90°: 90° + 90° + 90° + 90° = 360°. So, the figure is a rectangle. However, since the diagonals are perpendicular to each other, it must also be a rhombus and, more specifically, a square.

Challenge:

6. Student experimentation will vary. But the solution should be that the only two quadrilaterals that have diagonals that are perpendicular are a rhombus or a square.
ANSWERS:

1. **A** 30°

   The sum of the angle measures of a triangle must be 180°.
   
   \[ \text{50° + 100° = 150°} \]
   
   \[ 180° - 150° = 30° \]

2. In order to form a triangle, the angle measures must add to 180°. If they do, an infinite number of triangles can be constructed. All of the triangles will be similar to each other. This can be shown with parallel sides for the third side of a triangle.

3. In the first case, an infinite number of triangles can be formed because the angle between them can be anything, allowing the third side to be any length. Once that angle is given, there is only one side length for the third side.
4. The measure of angle 1 is 69°. The measure of angle 2 is 63°.
   Angle 2 is supplementary with the 117° angle because they form a straight line.
   \[180° – 117° = 63°\]

   The measures of three angles in the triangle must add to 180°. To find the measure of angle 1, add the two known angle measures and subtract from 180°.
   \[48° + 63° = 111°\]
   \[180° – 111° = 69°\]

   **Challenge Problem**

5. Since \(\triangle DEF\) is equilateral, each of its angles is congruent and must measure 60°.
   \[180° ÷ 3 = 60°, \text{ or } 3 \times 60° = 180°\]

   Angle \(EFG\) measures 120° because it forms a straight line with angle EFD.
   \[180° – 60° = 120°\]

   \(\triangle EFG\) is an isosceles triangle, so it has two congruent angles.
   Subtract 120° from 180° to find the sum of the congruent angle measures.
   \[180° – 120° = 60°\]

   Divide the sum by 2 to find the measure of each angle.
   \[60° ÷ 2 = 30°\]

   Angle \(DEG\) measures 90° because it is the sum of the two adjacent angles, angle \(DEF\) and angle \(FEG\).
   \[60° + 30° = 90°\]

   Since \(\triangle DEG\) has a 90° angle it is a right triangle. All of the side lengths are different so this is also a scalene triangle.
LESSON 6: PROPERTIES OF TRIANGLES

ANSWERS

1. **B** Right triangle
   - **D** Isosceles triangle

   Since two of the angles are 45º, the third angle must be 90º.
   
   \[45^\circ + 45^\circ + 90^\circ = 180^\circ\]

   This makes the triangle a right triangle. Since two of the angles are congruent, the triangle will also have two congruent sides, so it is also an isosceles triangle.

2. **C** Two of the sides must be the same length.

   If a triangle has two congruent angles, then it is an isosceles triangle.
   An isosceles triangle has two congruent sides.

3. **B** All three angles are the same measure.

   If a triangle has three congruent sides, then it is an equilateral triangle.
   An equilateral triangle has three congruent angles.

4. **D** The sum of the lengths of the two shorter sides must be greater than the length of the third side.

5. **A** 5 cm  
   **B** 13 cm  
   **C** 27 cm

   The three shorter lengths all work, 30 cm and 38 cm are too long to fit with these side lengths.
Challenge Problem

6. No, you cannot form a triangle with these sticks. Possible explanation:
The two shorter lengths, 5 cm and 7 cm add up to 12 cm. But the third stick is longer than that sum: 15 cm > 12 cm. Here is a diagram showing why these sticks cannot form a triangle.

![Diagram of sticks](image)

No matter how you arrange the three sticks, they cannot form a triangle. Imagine swinging points C and D to any position. They will never reach each other to complete the triangle.
1. **A 900°**

   There are 7 sides, so the polygon can be divided into 5 triangles.

   \[(n - 2)180°\]

   \[(7 - 2)180° = 5 \cdot 180° = 900°\]

2. **C Octagon**

   The angle sum for an octagon is 1,080°: \(8 - 2\)\(180° = 6 \cdot 180° = 1,080°\).

   Each of the 8 angles is congruent since it is a regular polygon: \(1,080° ÷ 8 = 135°\).

3. **B 130° and 140°**

   The angle sum for a pentagon is 540°: \(5 - 2\)\(180° = 3 \cdot 180° = 540°\).

   The sum of the measures of the 3 right angles is 270°: \(3 \cdot 90° = 270°\).

   Subtract 270° from 540° to find the measures of the two remaining angles:

   \(540° - 270° = 270°\). Answer choice B is the only choice that works: \(130° + 140° = 270°\).

   The two angles could be any combination that adds to 270° (except for 180°).

4. **No. Possible explanation:** Although the figure is divided into four triangles, the number of degrees has not changed and remains 360° for a quadrilateral. The four angles that meet at the center of the parallelogram are not part of the angles of the parallelogram. The sum of the measures of those angles is 360° because they make a complete circle. So, if the four triangle sums are added to get 720°, you must also subtract 360°: \(720° - 360° = 360°\).
LESSON 7: ANGLE SUMS

5. The angle sum is not correct because 2,400° is not evenly divisible by 180°. However, 2,340° is a multiple of 180° (13 • 180°), and the student could have easily made an accumulated error of 60° while measuring the 15 angles. The error would be about 4° per angle.

Challenge Problem

6. a. Possible answer: For each polygon, one triangle section can be analyzed since they are all the same. The triangles are the same because the diagonal segments are all congruent and the third sides, the polygon sides, are all congruent. This also means each triangle is an isosceles triangle and that the two angles at the polygon side are congruent. The angle measures in the polygons are known and any two angles at the polygon side add to equal this measure. The third angle at the center is found by subtracting the polygon angle measure from 180°:

   Square: 180° – 90° = 90°
   Hexagon: 180° – 120° = 60°
   Octagon: 180° – 135° = 45°

b. Possible answer: The angle at the center is getting smaller. It will get closer and closer to zero as the polygon angle gets closer and closer to 180°. The polygon would be getting closer to a circle and the segments going to the center are getting closer to being radii.
LESSON 8: FINDING ANGLES

ANSWERS:

1.  
   **Rhombus**
   
   All parallelograms have diagonals that divide the other diagonal in half. But, the diagonals of a rhombus are also perpendicular.

2.  
   **130°**
   
   The angle sum in a pentagon is 540°. Add the known angle measures. Then subtract that sum from 540° to find the measure of the remaining angle.
   
   \[95° + 105° + 90° + 120° = 410°\]
   
   \[540° - 410° = 130°\]

3.  
   \[x = 40°\]
   
   \[3x \text{ and } 60° \text{ are supplementary angles.} \]
   
   \[3x + 60° = 180°\]
   
   \[3x = 120°\]
   
   \[x = 40°\]

4.  
   \[x = 20°\]
   
   The angle measures of a triangle add to 180°.
   
   \[x + 2x + 6x = 180°\]
   
   \[9x = 180°\]
   
   \[x = 20°\]
5. a. The measure of angle 1 is 22.5°.
   Possible answer: The measure of each angle of a regular octagon is 135°.
   \[(8 - 2)180° = 6 \cdot 180° = 1,080°\]
   \[1,080 ÷ 8 = 135°\]
   Angle 1 is part of an isosceles triangle because the sides of a regular polygon are the same length. So, angle 1 and the other angle are congruent. Subtract 135° from 180° and divide by 2 to find the measure of each of the two congruent angles.
   \[(180° - 135°) ÷ 2 = 45° ÷ 2 = 22.5°\]

b. The figure is a square.
   Possible answer: The measure of each smaller angle of the isosceles triangle formed by the square and the octagon is 22.5°.
   \[22.5° + 90° + 22.5° = 135°\]
   Using the same process all around the octagon, each angle of the inside figure is 90°. Each side is the same length because they are part of congruent triangles. So, the figure inside is a square.

Challenge Problem

6. The measure of angle 1 is 75°. The measure of angle 2 is 15°. The measure of angle 3 is 15°. The measure of angle 4 is 150°.
   Possible answer: Since \(\triangle ABC\) is isosceles it has two congruent angles, so angle 1 is also 75°. The last angle in \(\triangle ABC\) must be 30° because 75° + 75° + 30° = 180°. This 30° angle and angle 3 total 45°, so angle 3 = 15° (45° - 30° = 15°). Angle 2 is also 15° since the triangle is isosceles. This leaves 150° for angle 4: 15° + 15° + 150° = 180°.